Red: Mo

Blue: Somil

Purple: Discuss

- The authors should probably reference the work of LaValle and that of Pallottino as they seem to be related to the subject of the paper. I would suggest:

LaValle, S. M., & Hutchinson, S. A. (1998). Optimal motion planning for multiple robots having independent goals. IEEE Transactions on Robotics and Automation, 14(6), 912-925.

Pallottino, L., Scordio, V. G., Bicchi, A., & Frazzoli, E. (2007). Decentralized cooperative policy for conflict resolution in multivehicle systems. IEEE Transactions on Robotics, 23(6), 1170-1183.

But there are several other papers by the authors that may be relevant.

Add these in the introduction

- By imposing a total order on the vehicles, the authors give up completeness. They should be clear about this in the paper and they should discuss why such a trade-ff is worthwhile. Note that saying that the problem is too difficult to solve otherwise is not a good reason. The authors should convince the reader that there are realistic scenarios where such an approach is well justified. Further, as Reviewer 1 observes, there should be some discussion on how the ordering can be chosen.

Our algorithm does not necessarily give up completeness, as in the trajectory planning problem statement, the initial time (called latest departure time) is a free variable that can always be found. This point is addressed in later comments in this response.

The ordering can be chosen in a first-come-first-serve basis by, for example, an air navigation service provider, as discussed in Reference [2]. Here, different agencies and companies can request air space-time, which is assigned in order of the time of request, or even in a batch with the ordering within the batch decided by the service provider. Although the deciding on an ordering is out of the scope of our paper, a brief discussion of this point is added in Section I-A and end of Section II.

- The authors should report the computational times for the simulations. Are the authors claiming that their approach can be used online? If yes, the simulations should convince the reader that is feasible. If not, the authors should explain how exactly will their approach benefit applications. In fact, the paper would be much stronger if the approach was demonstrated on real UAVs.

Almost all the computation is done offline, and involves computing the value functions associated with reachable sets. The gradient of the value function is stored as a look-up table, and accessed online in real-time to synthesize the controller through, for example, equations (13) or (19). For control-affine systems with bounded control, the optimal controls have an analytic form. We have clarified this point in Sections I-A and II. We have also added computation times in Sections IV-B, V-B, and VI-B.

We fully agree that hardware experiments would strengthen this paper, and are currently working on implementing our work in our quadrotor testbed.

- The authors claim that the algorithm scales linearly with the number of vehicles (with no intruder). However, it would appear that the number of obstacles is also important. As the complexity of obstacles (constraints) is on average n/2 for each vehicle, the computational complexity may actually grow quadratically. A discussion in the paper may be in order.

The algorithm indeed scales linearly with the number of vehicles (with no intruder). This is because we are using a single function in Equation (9) to capture all obstacles to be avoided by vehicle ; in implementation, this would be a single look-up table. For trajectory planning for the next vehicle , can simply be updated by adding an additional term to the union in Equation (9).

It is true that Step 4 of algorithm 1 involves computing for all , which makes the complexity technically quadratic. However, this computation simply involves storing in a space that contains all vehicles – if the state spaces of the vehicles are at most 4 dimensional in , then is a 4D look-up table – and then updated recursively via . Then, during trajectory planning, vehicle would simply ignore any irrelevant dimensions of .

We have clarified this point at the end of Section IV-A.

Reviewer 2 suggests that it may be worthwhile directly comparing the approach with a more recent work in the literature on multi-agent systems.

Finally, Reviewer 3 suggests how to improve the literature review and the bibliography. The reviewer also asks for a more extensive simulation and challenges the second assumption in Section VI.

Reviewer: 1. Comments to the Author

The considered problem is timely and strongly motivated, especially in the context of recent advances in the use of UAVs. The main idea is in fact very simple, but that is in my opinion what makes it elegant and applicable. It might be interesting though to include a discussion on how the ordering of the vehicles is determined and how it influences the outcome, i.e. the latest departure times of the vehicles. For instance, in the numerical simulations, the chosen ordering is in accordance with the growing scheduled time of arrival. Is it a coincidence or is there a particular reason for that?

Towards end of Section II and one subsequent section: Clarify this is not a coincidence, and it’s not chosen!

Minor comments:

- I would appreciate a little bit more details and intuitive explanations on time-varying reachability background.

- Beginning of Sec. IV: “… SPP algorithm can still serve as a useful approximation in certain situations”: It might be helpful to give a concrete example here.

- “This is precisely the set P\_i(t).”: Please clarify.

- It might be helpful to add a discussion on how the reduced control set and the parameter R\_EB should be chosen in Sec. V.A.3.

- It may be interesting to add a clarification of the following questions: In general, avoiding the intruder may cause a vehicle not being able to reach a target region any more, correct? Does the optimal avoidance control from Eq. (55) ensures the target region remains reachable?

Reviewer: 2. Comments to the Author

1. In section 3, the backward and forward reachable set are introduced and computed by solving the partially differential equations, i.e., Eq.~(5) and Eq.~(6), in which $D\_t$ is not formally given.

2. In this paper, authors define the danger zone $Z\_{i,j}=\{(x\_i,x\_j):||p\_i-p\_j||\_2\leq R\_c\}$ only based on the position information. In reality applications, there exists a kind of inevitable danger zone, where the robot or UAV will be in danger zone with a hundred percent. It is suggested to provide more details about such inevitable danger zones.

Section II and below Eq. (11): We need to say given these danger zones, backward reachability by definition avoids “inevitable danger zones”

3. Authors proposed a sequential planning method in order to make the reachable set method computing trackable with different priority when facing high dimensional problems. It makes sense in real applications that each agent has different priority. However, it may return failure when priority is not crucial or even can be ignored and the inappropriate priority is given. If the priority issue has been considered by authors, please give more details and explanation.

This is a good point, and requires clarification on our part.

Our algorithm always returns a safe and timely feasible trajectory, as long as one exists in the absence of other vehicles. Feasibility is also independent of the chosen priority. This is because any vehicle can simply depart early enough to avoid crowded environments to arrive earlier than required. The latest time of arrival provided by our algorithm quantifies exactly how early each vehicle needs to depart given a priority ordering. Of course, practically speaking there may be situations in which a vehicle cannot depart early enough; however, this is always a concern with any trajectory planning method.

We have clarified this point near the end of Section II, and at the end of Section IV-A.

4. Comparative studies are suggested to show the advantages of the proposed algorithm. In addition, more recent work of multi-agent (AGV, UAV, AUV, etc) is suggested to be mentioned in the introduction.

Reviewer: 3

in section I :

The references are presented succinctly. Make one sentence per reference should help the reader to understand issues and contributions.

Some references should probably be removed. (see "1-5", "6:7", "8-11", "17-19", ...)

Some references are cited as other contributions, called "and many others" by Authors (see "23-25", "37-38", ...)

Revise references in introduction?

in section II, III, IV, V :

Vehicules seems to be particules ?

Make one clarifying sentence about danger zone includes effective size

Configuration spaces is then simplified and robustness is delegated to surrounding distance of other things.

time-varying reachability is formalized

perfect and imperfect information cases are presented with numerical simulations for four vehicules.

comparisons to previous works are missing

simulations should involve more vehicules and more obstacles

in section VI :

The case of an environment with a single moving obstacle is presented as intruder or adversarial vehicules.

The second assumption is an unrealistic condition.

Add phrase to acknowledge this

in bibliography :

page number is sometimes missing

reference title is sometimes written with capitals (see 12, 29)

Out of fourty-three references, only fourteen have been published after 2010. It confirms that some references are not needed.

Revisit references

Other changes:

We have changed the phrase “sequential path planning” to “sequential *trajectory* planning” to emphasize that our proposed algorithms produce a continuous sequence of states over time, as opposed to just a geometric curve in space.